Graph Watermarks

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ABSTRACT

From network topologies to online social networks, many of today's most sensitive datasets are captured in large graphs. A significant challenge facing owners of these datasets is how to share sensitive graphs with collaborators and authorized users, *e.g.* network topologies with network equipment vendors or Facebook's social graphs with academic collaborators. Current tools can provide limited node or edge privacy, but require modifications to the graph that significantly reduce its utility.

In this work, we propose a new alternative in the form of graph watermarks. Graph watermarks are small graphs tailor-made for a given graph dataset, a secure graph key, and a secure user key. To share a sensitive graph G with a collaborator C, the owner generates a watermark graph Wusing G, the graph key, and C's key as input, and embeds W into G to form G'. If G' is leaked by C, its owner can reliably determine if the watermark W generated for C does in fact reside inside G', thereby proving C is responsible for the leak. Graph watermarks serve both as a deterrent against data leakage and a method of recourse after a leak. We provide robust schemes for creating, embedding and extracting watermarks, and use analysis and experiments on large, real graphs to show that they are unique and difficult to forge. We study the robustness of graph watermarks against both single and powerful colluding attacker models, then propose and empirically evaluate mechanisms to dramatically improve resilience.

1. INTRODUCTION

Many of today's most sensitive datasets are captured in large graphs. Such datasets can include maps of autonomous systems in the Internet, social networks representing billions of friendships, or connected records of patent citations. Controlling access to these datasets is a difficult challenge. More specifically, it is often the case that owners of large graph datasets would like to share access to them to a fixed set of entities without the data leaking into the public domain. For example, an ISP may be required to share detailed network topology graphs with a third party networking equipment vendor, with a strict agreement that access to these sensitive graphs must be limited to authorized personnel only.

Similarly, a large social network like Facebook or LinkedIn may choose to share portions of its social graph data with trusted academic collaborators, but clearly want to prevent their leakage into the broader research community.

One option is to focus on building strong access control mechanisms to prevent data leakage beyond authorized parties. Yet in most scenarios, including both examples above, data owners cannot restrict physical access to the data, and have limited control once the data is shared with the trusted collaborator. It is also the case that no matter how well access control systems are designed, they are never foolproof, and often fall prey to attacks on the human element, *i.e.* social engineering. Another option is to modify portions of the data to reduce the impact of potential data leakages. This has the downside of making the data inherently noisy and inaccurate, and still can be overcome by data reconstruction or de-anonymization attacks using external input [27]. Finally, these schemes are hard to justify, in part because it is very difficult to quantify the level of protection they provide.

In this work, we propose a new alternative in the form of graph watermarks. Intuitively, watermarks are small, often imperceptible changes to data that are difficult to remove, and serve to associate some metadata to a particular dataset. They are used successfully today to limit data piracy by music vendors such as Apple and Walmart, who embed a user's personal information into a music file at the time of purchase/download [3]. Should the purchased music be leaked onto music sharing networks, it is easy for Apple to track down the user who was responsible for the leak. In our context, graph watermarks work in a similar way, by securely identifying a copy of a graph with its "authorized user." Should a shared graph dataset be leaked and discovered later in public domains (on BitTorrent perhaps), the data owner can extract watermark from the leaked copy and use it as proof to seek damages against the collaborator responsible for the leak. While not a panacea, graph watermarks can provide additional level of protection for data owners who want to or must share their data, and perhaps encourage risk-averse data owners to share potentially sensitive graph data, e.g. encourage LinkedIn to share social graphs with academic collaborators.

To be effective, a graph watermark system needs to pro-

vide several key properties. First, graph watermarks should be relatively small compared to the graph dataset itself. This has two direct consequences: the watermark will be difficult to detect (and remove) by potential attackers, and adding the watermark to the graph has minimal impact on the graph structure and its utility. Second, watermarks should be difficult to forge and should not occur naturally in graphs, ensuring that the presence of a valid watermark can be securely associated with some user, i.e. non-repudiation. Third, both the embedding and extraction of watermarks should be efficient, even for extremely large graph datasets with billions of nodes and edges. Finally, our goal is to design a watermark system that works in any application context involving graphs. Therefore, we make no assumptions about the presence of metadata. Instead, our system must function for "barebones" graphs, i.e. symmetric, unweighted graphs with no node labels or edge weights.

In this paper, we present initial results of our efforts towards the design of a scalable and robust graph watermark system. Highlights of our work can be organized into the following key contributions.

- First, we identify the goals and requirements of a graph watermark system. We also describe an initial design of a graph watermark system that efficiently embeds watermarks into and extracts watermarks out of large graphs. Graph watermarks are uniquely generated based on a user private key, a secure graph key, and the graph they are applied to. We describe constraints on its applicability, and identify examples of graphs where watermarks cannot achieve desirable levels of key properties such as uniqueness.
- Second, we provide a strict proof of uniqueness of graph watermarks, showing that it is extremely difficult for attackers to forge watermarks.
- Third, we evaluate our watermarks in term of distortion, false positive, and efficiency on a wide variety of large graph datasets.
- Fourth, we identify two attack models, describe additional features to boost robustness, and evaluate them under realistic conditions.

To the best of our knowledge, our work is the first practical proposal for applying watermarks to graph data. We believe graph watermarks are a useful tool suitable for a wide range of applications from tracking data leaks to data authentication. Our work identifies the problem and defines an initial groundwork, setting the stage for follow-up work to improve robustness against a range of stronger attacks.

2. BACKGROUND AND RELATED WORK

In this section, we provide background and related work on the graph privacy problem and discuss the use of watermark techniques in applications such as digital multimedia as well as graphs. **Graph Privacy.** Graph privacy is a significant problem that has been magnified by the arrival of large graphs containing sensitive data, *e.g.* Facebook social graphs or mobile call graphs. Recent studies [4, 27] show that deanonymization attacks using external data can defeat most common anonymization techniques.

A variety of solutions have been proposed, ranging from anonymization tools that defend against specific structural attacks, or more attack-agnostic defenses. To protect nodeor edge-privacy against specific, known attacks, techniques utilize variants of k-anonymization to produce structural redundancy at the granularity of subgraphs, neighborhoods or single nodes [23, 46, 11, 47]. Alternatively, randomization provides privacy protection by randomly adding, deleting, or switching edges [10, 45]. Others partition the nodes and then describing the graph at the level of partitions to avoid structural re-identification [12]. Finally, other solutions have taken a different approach, by producing model-driven synthetic graphs that replicate key structural properties of the original graph [36]. One extension of this work utilizes differential privacy techniques to provide a tunable accuracy vs. privacy tradeoff [37].

The goals of our work are quite different from prior work on graph anonymization, meant to protect data before its public release. We are concerned with scenarios where graph data is shared between its owner and groups of trusted collaborators, *e.g.* third party network vendors analyzing an ISP's network topology, or Facebook sharing a graph with a small set of academic researchers. The ideal goal in these scenarios is to ensure the shared data does not leak into the wild. Once data is shared with collaborators, reliable tools that can track leaked data back to its source serve as an excellent deterrent. Watermarking techniques have addressed similar problems in other contexts, and we briefly describe them here.

Background on Digital Watermarks. Watermarking is the process of embedding specialized metadata into multimedia content such as images or audio/video files [14]. This embedded watermark is later extracted from the file and used to identify the source or owner of the content. These systems include both an embedding component and an extraction or recovery component. The embedding component takes three inputs: a watermark, the original data, and a key. The watermark is embedded into the data in a way that minimizes impact on the data, i.e. transparent letters overlaid on top of an image. The key is used as a parameter to change the way the watermark is embedded, usually corresponds to a specific user, and is kept confidential by the data owner to prevent unauthorized parties from recovering and modifying the watermark. Extraction takes as input the watermarked data, the key, and possibly a copy of the original data. Extraction can directly produce the embedded watermark or a confidence measure of whether it is present.

Significant work has been done in digital watermarking, particularly image watermarking [38, 24, 5, 35, 43]. Image

watermarking techniques can be classified into two classes based on their working domains. The first class of watermarks is applied to the original domain of the image, the spatial domain. Basic techniques include modifying the least significant bits of each image pixel on the original image to encode the watermark [38, 24, 5]. The second class applies watermarks to the transformed domain of the image, *i.e.* the frequency domain. The original data is first transformed into frequency domain using DCT [31], DFT [35] or DWT [43], added a sequence of small noises to several invisible frequencies, and then the result is transformed back into spatial domain as the watermarked image. The sequence of noises is the watermark, and can be extracted by carrying out the reverse process on the watermarked image.

Watermark techniques are already widely used today to protect intellectual property. Watermark techniques [29, 30] have been studied to protect the abuse of digital vector maps. Like image watermarks, these techniques can be classified as spatial domain methods and transformed domain methods. Unlike image watermarks, the spatial domain methods embed watermarks by modifying vertex coordinates [29], while the transformed domain methods tend to transform vector maps into a different frequency domain, such as the meshspectral domain [30]. Watermarks have also been used to protect software copyrights, by adding spurious execution paths in the code that would not be triggered by normal inputs [7, 40]. These execution paths are embedded as extra control flows between blocks of code, and are triggered (or extracted) by either locating the code, or running the program with a special input that triggers the alternate execution paths. Moreover, algorithms have been proposed for watermarking relational datasets [1, 22, 13]. Much of this has focused on modifying numeric attributes of relations, relying on the primary key attribute as indicator of watermark locations, assuming that the primary key attribute does not change. Finally, watermarks, in the form of minute changes, have been applied to protect circuit designs in the semiconductor industry [32, 42].

3. GOALS AND ATTACK MODELS

To set the context for the design of our graph watermark system, we need to first clearly define the attack models we target, and use them to guide our design goals.

Graph watermarks at a glance. At a high level, we envision the graph watermark process to be simple and lightweight, as pictured in Figure 1. Embedding a watermark involves *overlaying* the original graph dataset (G) with a small subgraph (W) generated using the original graph and a secret random generator seed (Ω) . Embedding the watermark simply means adding or deleting edges between existing nodes in the original graph G, based on the watermark subgraph W. Each authorized user i receives only a watermarked graph customized for them, generated using a random seed Ω_i securely associated with her. The seed is generated through cooperation of her private key and a key

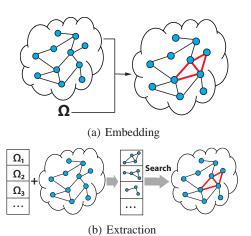


Figure 1: Embedding and extracting graph watermarks. Ω is a secret random generator seed produced using the secure graph key and user's private key.

securely associated with the original graph.

If and when the owner detects a leaked version of the dataset, the owner takes the leaked graph, and "extracts the watermark," by iteratively producing all known watermark subgraphs W_i associated with G and each of the seeds Ω_i associated with an authorized user. The "extraction" process is actually a matching process where the data owner can conclusively identify the source of the leaked data, by locating the matching W_i in the leaked graph.

In our model of potential attackers and threats, we assume that attackers have access to the watermarked graph, but not the original G. Clearly, if an attacker is able to obtain the unaltered G, then watermarks are no longer necessary or useful.

Attack Models. The attackers' goal is to destroy or remove graph watermarks while preserving the original graph. Watermarks are designed to protect the overall integrity of the graph data. Thus we do not consider scenarios where the attackers sample the graph or distort it significantly in order to remove the watermark. Doing so would be analogous to removing a portion of all pixels from a watermarked video, or applying a high pass frequency filter to watermarked music. Under these constraints, we consider two practical attack models below.

- Single Attacker Model. For a single attacker with access to one watermarked graph, it will be extremely difficult to detect the watermark subgraph. Without the key associated with another user, forging a watermark is also impractical. Instead, their best attack is to disrupt any potential watermarks by making modifications, i.e. add or delete nodes or edges.
- Collusion Attack Model. If multiple attackers join their efforts, they can recover the original graph by comparing multiple watermarked graphs, identifying the differences (i.e. watermarks), and removing them.

Design Goals. These attack models help us define the key characteristics required for an effective graph watermarking system.

- *Low distortion*. The addition of watermarks should have a small impact on overall structure of the original graph. This preserves the utility of the graph datasets.
- Robust to modifications. Watermarks should be robust to modification attacks on watermarked graphs, i.e. watermarks should remain detectable and extractable with high probability, even after the graph has been modified by an attacker.
- Low false positives. It is extremely unlikely for our system to successfully identify a valid watermark W_i in an unwatermarked graph or a graph watermarked by W_j where $i \neq j$. When we embed a single watermark (Section 4), we also refer to this property as watermark uniqueness.

Within the constraints defined above, designing a graph watermark system is quite challenging, for several reasons. First, the subgraph that represents the watermark must be relatively "unique," i.e. it is highly unlikely to occur naturally, or intentionally through forgery. A second, contrasting goal is that the watermark should not change the underlying graph significantly (low distortion), or be easily detected. Walking the fine line between this and properties of "uniqueness" likely means we have to restrict the set of graphs which can be watermarked, i.e. for some graphs, it will be impossible to find a hard to detect watermark that does not occur easily in graphs. Finally, since any leaked graph can have all metadata stripped or modified, watermark embedding and extraction algorithms must function without any labels or identifiers. Note that the problem of subgraph matching is known to be NP-complete [8].

4. BASIC WATERMARK DESIGN

We now describe the basic design of our graph watermarking system. The basic design seeks to embed and extract watermarks on graphs to achieve watermark uniqueness while minimizing distortion on graph structure. Our design has two key components:

- Watermark embedding: The data owner holds a graph key K^G associated with a graph G known only to her. Each user i generates its public-private cryptographic key pair $< K^i_{pub}, K^i_{priv} >$ through a standard public-key algorithm [25], where K^i_{pub} is user i's public key and K^i_{priv} is its corresponding private key. To share the graph G with user i, the system combines input from user i digital signature $K^i_{priv}(T)$ and graph key K^G to form a random generator seed Ω_i , and use Ω_i to generate a watermark graph W_i for graph G. The system embeds W_i into G by selecting and modifying a subgraph of G that contains the same number of nodes as W_i . The resulting graph G^{W_i} is given to user i as the watermarked graph.
- Watermark extraction: To identify the watermark in G',

we use Ω_i to regenerate W_i and then search for the existence of W_i within G', for each user i.

In this section, we focus on describing the detailed procedure of these two components. We present detailed analysis on the two fundamental properties of graph watermarks, *i.e.* uniqueness and detectability in Section 5.

4.1 Watermark Embedding

The most straightforward way to embed a watermark is to directly attach the watermark graph to the original graph. That is, if W_i represents the watermark graph for user i, and G represents the original graph to be watermarked, the embedding treats W_i as an independent graph, and adds new edges to connect W_i to G. However, this approach has two disadvantages. First, direct graph attachment makes it easy for external attackers to identify and remove W_i from G without using graph key K^G and user i's signature $K^i_{priv}(T)$. New edges connecting W_i and G must be carefully chosen to reduce the chance of detection, and this is a very challenging task. Second, attaching a (structurally different) subgraph W_i directly to a graph G introduces larger structural distortions.

Instead, we propose an alternative approach that embeds the watermark graph "in-band." That is, the embedding process first selects k nodes (k is the number of nodes in W_i) from G and identifies S, the corresponding subgraph of G induced by these k nodes. It then modifies S using W_i without affecting any other nodes in G. Because the watermark graph W_i is naturally connected with the rest of the graph, both the risk of detection and amount of distortion induced on the original graph G are significantly lower than those of the direct attachment approach.

We now describe the details of "in-band" watermark embedding, which consists of four steps: (1) generating random generator seed Ω_i from user i's signature $K^i_{priv}(T)$ and graph key K^G ; (2) generating the watermark graph W_i from the seed Ω_i ; (3) selecting the placement of W_i on G by picking k nodes from G and identifying the corresponding subgraph S induced by these k nodes; and (4) embedding W_i into G by modifying S to match structure of W_i .

Step 1: Generating random generator seed Ω_i . To generate an unforgetable watermarked graph, we generate a random generator seed Ω_i [9] using user i's signature $K^i_{priv}(T)$ and graph key K^G .

Suppose the system intends to generate a watermarked version of graph G at time T to share with a specific user i. We begin by first sending user i with the current timestamp T. User i responds with its signature $K^i_{priv}(T)$, by encrypting the timestamp with its private key K^i_{priv} . Before proceeding further, we validate the result $K^i_{priv}(T)$ to ensure it is from i, by decrypting it with user i's public key K^i_{pub} . If the timestamps match, we combine the signature $K^i_{priv}(T)$ and the graph key K^G to form the seed of the random graph generator for user i, Ω_i . A mismatch may indicate that user

i is a potential malicious user.

Note that Ω_i cannot be formed alone by the data owner who only holds the graph key K^G , or by user i who only owns its private key K^i_{vriv} . Therefore, results computed using seed Ω_i , including the random graph W_i generated (Step 2) and the choice of graph nodes to mark (Step 3), cannot be derived independently by the data owner or identified by user i.

Step 2: Generating the watermark graph W_i . We generate W_i as a random graph with edge probability of p and node count k (k << n where n is the number of nodes in G). The random edge generator uses Ω_i as the seed [9]. The k nodes of W_i are ordered as $\{v_1, v_2, ..., v_k\}$.

The key factor in this step is choosing the node count k and the edge probability p. As we will show in Section 5.1, the two parameters must satisfy the following requirement to ensure watermark uniqueness:

$$k \ge (2 + \delta) \log_a n \tag{1}$$

 $k \geq (2+\delta)\log_q n \tag{1}$ where $q=\frac{1}{\max{(p,1-p)}}$ and δ is a constant >0. Furthermore, it is easy to prove that $p=\frac{1}{2}$ minimizes the node count k and the average edge count $p \cdot \binom{k}{2}$ of the watermark graph W_i . Intuitively, using a compact watermark graph not only reduces the amount of distortion to G, but also improves its robustness against malicious attacks. Therefore, we configure $p=\frac{1}{2}$ and therefore $k=(2+\delta)\log_2 n$. This produces a reasonably sized watermark graph (k < 100) even for extremely large graphs, e.g. the complete Facebook social graph (\sim 1 billion nodes in 2014).

Step 3: Selecting the watermark placement on graph G. Next, we identify k nodes from G and its corresponding subgraph S to embed the watermark graph. To ensure reliable extraction, we must choose these k nodes carefully, meeting these two requirements. First, using Ω_i generated in Step 1, the k nodes must be chosen deterministically and remain distinguishable from the other nodes of G. Second, the set of the k nodes chosen for different watermarks (or different Ω_i values) must be easily distinguishable from each other to reinforce watermark uniqueness. Our biggest challenge in meeting these requirements is that we cannot use node IDs to distinguish nodes from each other. Node IDs or any type of metadata can be easily altered or stripped by attackers before or after leaking G', thereby making extraction impossible.

We address this challenge by using local graph structure around each node as its "label." Specifically, we define a node structure description (NSD) as a distinguishable feature of each node. A node v's NSD is represented by an array of v's sorted neighbor degrees. For example, if node vhas three neighbors with node degrees 2, 6, 4, respectively, then v's NSD label is "2-4-6." We then hash v's NSD label into a numerical value using a secure one-way hash e.g. SHA-1 [34], and refer to the result as node v's NSDhash.

Next, we use Ω_i as the seed to randomly generate k hash values, and use each as an index (e.g. using a mod function) to identify a node in G. It is possible that multiple nodes have the same NSDhash, i.e. a collision. If this happens, we resolve the collision by using Ω_i again as an index into a sorted list of these nodes with the same NSDhash. The nodes can be sorted by any deterministic order, e.g. node IDs in the original graph. Note that this process is only required for embedding (and not extraction), so any deterministic order chosen by the graph owner will suffice.

At the end of this step, we obtain k ordered nodes from $G, X = \{x_1, x_2, ..., x_k\},$ and the corresponding subgraph S = G[X] induced by the node set X on G.

Step 4: Embedding the watermark graph W_i into graph In this step, we embed the watermark graph W_i by modifying the subgraph S = G[X] to match W_i . Specifically, we match each (ranked) node in W_i , $\{v_1, v_2, ..., v_k\}$ with the corresponding node in S (or X), $\{x_1, x_2, ..., x_k\}$, i.e. $f: W \to S, f(v_i) = x_i$. And once the nodes are mapped, we then apply an XOR operation on each edge of the two graphs. That is, we consider the connection between (v_i, v_j) or (x_i, x_j) as one bit, i.e. an edge between (v_i, v_j) or (x_i, x_j) means 1 and no edge between (v_i, v_j) or (x_i, x_j) means 0. If an edge (v_i, v_j) exists in W_i , we modify the corresponding edge value in S from (x_i, x_j) to $(x_i, x_j) \oplus 1$; and if no edge (v_i, v_j) exists in W_i , we modify the edge value (x_i, x_j) to $(x_i, x_j) \oplus 0$. When the above edge modification process ends, we also explicitly create edges between nodes x_i and x_{i+1} to maintain a connected subgraph. As a result, we transfer the subgraph S into S^{W_i} with the watermark graph W_i embedded. The reason for choosing the XOR operation is that it allows the same watermark to be embedded in the graph multiple times (at multiple locations), thus reducing the risk of the watermark being detected and destroyed by attacks such as frequent subgraph mining. We will discuss this in more details in Section 6.

At the end of this step, we obtain a watermarked graph G^{W_i} for user i. Before we distribute it to user i, we anonymize G^{W_i} by completely (randomly) reassigning all node IDs. Such anonymization not only helps to protect user privacy, but also minimizes the opportunity for colluding attackers with multiple watermarked graphs to identify the embedded watermark (see Section 6).

4.2 **Watermark Extraction**

The watermark extraction process determines if a watermark graph W_i is embedded in a target graph G'. If so, then G' is a legitimate copy distributed to user i. The extraction process faces two key challenges. First, the target graph G', likely a watermarked version of the original graph G, can easily be modified by users/attackers during the graph distribution process. In particular, all node IDs can be very different from that of the original G. Thus extraction cannot rely on node IDs in G'. Second, identifying whether a subgraph exists in a large graph is equivalent to a subgraph matching problem, known to be NP-complete. To handle massive graphs, we need a computationally efficient algorithm.

Our design addresses these two challenges by leveraging

knowledge on the structure of the subgraph where the watermark was embedded. This eliminates the dependency on node IDs while significantly reducing the search space during the subgraph matching process. We describe our proposed design in detail below.

Step 1: Regenerating the watermark. The owner performs the extraction, and has access to the original graph G, graph key K^G , and user's signature $K^i_{priv}(T)$. For each user i, we combine its signature $K^i_{priv}(T)$ and graph key K^G to generate the random generator seed Ω_i for that user. Then, we follow step 2-4 described in Section 4.1 to regenerate the watermark graph W_i , identify the k ordered nodes from G and their NSD labels, and finally the modified subgraph S^{W_i} that was placed on a "clean" version of the watermarked graph G^{W_i} .

Step 2: Identifying candidate watermark nodes on G'. Given the k nodes $X = \{x_1, x_2, ..., x_k\}$ identified from the original graph G, in this step we need to identify for each x_i , a set of candidate nodes on the target graph G' that can potentially become x_i . We accomplish this by identifying all the nodes on G' whose NSD labels are the same of x_j in the "clean" version of the watermarked graph G^{W_i} . Since multiple nodes can have the same NSD label, this process will very likely produce multiple candidates. To shrink the candidate list, we examine the connectivity between candidate nodes of X on G' and compare it to that among X on G^{W_i} . If two nodes x_m and x_n are connected in G^{W_i} , we prune their candidate node lists by removing any candidate node of x_m that has no edge with any candidate node of x_n on G' and vice versa. This pruning process dramatically reduces the search space. After this step, we obtain for each x_i the candidate node list C_i on the target graph G'.

Step 3: Detecting watermark graph S^{W_i} on G'. Given the candidate node list of each node in X, we now search for the existence of S^{W_i} on the target graph G'. For this we apply a recursive algorithm to enumerate and prune the combinations of the candidate sets, until we identify S^{W_i} or exhaust all the node candidates. The detailed algorithm is listed in Algorithm 1. In this algorithm, we use a node list Y to record the list of nodes in G' which we have already finalized as the corresponding nodes in S^{W_i} , i.e. $Y = \{y_1, y_2, ..., y_m\}$ $(m \le k)$. When the process starts, $Y = \emptyset$, m = 0.

Discussion. The above design shows that our watermark extraction algorithm simplifies the subgraph search problem by restricting it to a small number of selected nodes from a graph, thus avoiding the NP-complete subgraph matching problem. Also note that we target real graphs with very high levels of node heterogeneity, *e.g.* small-world, power-law or highly clustered graphs, which are very far from the uniform, lattice-like graphs that are the worst case scenarios for graph isomorphism. In practice, our system can efficiently extract watermarks from real, million-node graphs, and do so in a few minutes on a single commodity server (Section 7.3).

Algorithm 1 Recursive Algorithm for Detecting S^{W_i} on G'.

```
1: Function: SubgraphDetection(G', S^{W_i}, \{C_1, C_2, ..., C_k\}, Y, m)
2: Input: Graph G', watermark graph S^{W_i}, candidate node list C_i for each node x_i in X, identified node list Y = \{y_1, y_2, ..., y_m\} (m < k)
3: Output: Identified node list Y
4: for each node c \in C_{m+1} do
         if c \notin Y and each edge (c, y_t) in G' (t = 1..m) is the same as the
         edge (x_{m+1}, x_t) in S^{W_i} (t = 1..m) then
6:
7:
             m = m + 1
8:
             if m == k then
9:
10:
11:
                  SubgraphDetection(G', S^{W_i}, \{C_1, C_2, ..., C_k\}, Y, m)
12:
              end if
13:
              Y = Y \setminus c
14:
              m = m - 1
15:
          end if
16: end for
17: Return Y
```

5. FUNDAMENTAL PROPERTIES

Having described the basic watermark system, we now present detailed analysis on its two fundamental properties: $watermark\ uniqueness$ where each watermark must be unique to the corresponding user, and $watermark\ detectability$ where the presence of a watermark should not be easily detectable by external users without the knowledge of the seed Ω_i associated with user i.

5.1 Watermark Uniqueness

As a proof of ownership, each embedded watermark should be unique for its user. That is, given the original graph G and the seed Ω_i associated with user i, the embedded watermark graph S^{W_i} should not be isomorphic to any subgraph of G^{W_j} ($i \neq j$) where G^{W_j} is the watermarked graph for user j. At the same time, S^{W_i} should not be isomorphic to any subgraph of the original graph G. In the following, we show that with high probability, our proposed graph watermark system produces unique watermarks for any graph G.

THEOREM 1. Given a graph G with n nodes, let $k \ge (2+\delta)\log_2 n$ for a positive constant $\delta > 0$. We apply the following process to create a watermarked graph G^{W_i} for user i:

- We create k nodes, $V = \{v_1, v_2, ..., v_k\}$, and generate a random graph W_i on V with an edge probability of $\frac{1}{2}$.
- We randomly select k nodes, $X = \{x_1, x_2, ..., x_k\}$ from G, and identify the subgraph corresponding to these k nodes S = G[X].
- Using W_i , we modify S as follows: we first map each node x_i in X to a node v_i in V. Let e(u,v)=1 denote an edge exists between node u and v and e(u,v)=0 denote otherwise. We modify each $e(x_i,x_j)$ in S to $e(x_i,x_j) \oplus e(v_i,v_j)$. We then explicitly connect nodes x_i and x_{i+1} , i.e. $e(x_i,x_{i+1})=1$. The resulting S now becomes S^{W_i} ,

Table 1: Suitability of watermarking for 48 of today's network graphs, determined by comparing their node degree distribution $[N_{min}(G), N_{max}(G)]$ and k-node subgraph density $[D_{min}(k), D_{max}(k)]$ to those of the embedded watermark

graphs. 35 out of these 48 graphs are suitable for watermarking.

Taplis. 33 out of these 40 graphs are suitable for watermarking.										
Graph	Graph	# of Nodes	# of Edges	Avg. Deg.	k	k Node Degree Criterion		k-node Subgraph Density Criterion		Suitability
Category	•					(k+1)/2	$[N_{min}(G), N_{max}(G)]$	Watermark	$[D_{min}(k), D_{max}(k)]$,
	Russia	97,134	289,324	6.0	39	20	[1, 748]	390	[45, 701]	Yes
Facebook	L.A.	603,834	7,676,486	25.4	45	23	[1, 2141]	517	[44, 975]	Yes
	London	1,690,053	23,084,859	27.3	48	24	[1, 1483]	588	[47, 1128]	Yes
	Epinions (1)	75,879	405,740	10.7	38	19	[1,3044]	370	[47,649]	Yes
	Slashdot (08/11/06)	77,360	507,833	13.1	38	19	[1, 2540]	370	[38, 668]	Yes
	Twitter	81,306	1,342,303	33.0	38	19	[1, 3383]	370	[44, 703]	Yes
Other	Slashdot (09/02/16)	81,867	497,672	12.2	38	19	[1, 2546]	370	[38, 669]	Yes
Social	Slashdot (09/02/21)	82,140	500,481	12.2	38	19	[1, 2548]	370	[38, 669]	Yes
Networks	Slashdot (09/02/22)	82,168	543,381	13.2	38	19	[1, 2553]	370	[38, 673]	Yes
	GPlus	107,614	12,238,285	227.5	39	20	[1, 20127]	389.5	[53, 741]	Yes
	Epinions (2)	131,828	711,496	10.8	40	20	[1, 3558]	409.5	[51, 780]	Yes
	Youtube	1,134,890	2,987,624	5.3	47	24	[1, 28754]	563.5	[47, 815]	Yes
	Pokec	1,632,803	22,301,964	27.3	48	24	[1, 14854]	587.5	[47, 979]	Yes
	Flickr	1,715,255	15,555,041	18.1	48	24	[1, 27236]	588	[51, 1128]	Yes
	Livejournal	5,204,176	48,942,196	18.8	52	26	[1, 15017]	689	[51, 1326]	Yes
Citation	Patents	23,133	93,468	8.1	34	17	[1, 280]	297	[37, 373]	Yes
Networks	ArXiv (Theo. Cit.)	27,770	352,304	25.4	34	17	[1, 2468]	297	[36, 534]	Yes
	ArXiv (Phy. Cit.)	34,546	420,899	24.4	35	18	[1, 846]	314.5	[36, 544]	Yes
	ArXiv (Phy.)	12,008	118,505	19.7	32	16	[1, 491]	263.5	[45, 496]	Yes
Collaboration	ArXiv (Astro)	18,772	198,080	21.1	33	17	[1, 504]	280	[37, 528]	Yes
Networks	DBLP	317,080	1,049,866	6.6	43	22	[1,343]	472.5	[43,903]	Yes
	ArXiv (Condense)	3,774,768	16,518,947	8.8	51	26	[1, 793]	663	[50,1063]	Yes
Communication	Email (Enron)	36,692	183,831	10.0	35	18	[1,1383]	314.5	[43,515]	Yes
Networks	Email (Europe)	265,214	365,025	2.8	42	21	[1,7636]	451	[74,683]	Yes
	Wiki	2,394,385	4,659,565	3.9	49	25	[1, 100029]	612	[65, 1066]	Yes
	Stanford	281,903	1,992,636	14.1	42	21	[1,38625]	451	[66,861]	Yes
Web	NotreDame	325,729	1,103,835	6.8	43	22	[1,10721]	472.5	[60,903]	Yes
graphs	BerkStan	685,230	6,649,470	19.4	45	23	[1,84230]	517	[79,990]	Yes
	Google	875,713	4,322,051	9.9	46	23	[1, 6332]	540	[72, 1033]	Yes
Location based	Brightkite	58,228	214,078	7.4	37	19	[1,1134]	351	[41,665]	Yes
OSNs	Gowalla	196,591	950,327	9.7	41	21	[1,14730]	430	[44,723]	Yes
	Oregon (1)	11,174	23,409	4.2	31	16	[1,2389]	247.5	[95,352]	Yes
AS	Oregon(2)	11,461	32,730	5.7	32	16	[1,2432]	263.5	[79,476]	Yes
Graphs	CAIDA	26,475	53,381	4.0	34	17	[1,2628]	297	[113,436]	Yes
1	Skitter	1,696,415	11,095,298	13.1	48	24	[1, 35455]	588	[52, 1128]	Yes
	Gnutella (02/08/04)	10,876	39,994	7.4	31	16	[1,103]	247.5	[30,80]	No
	Gnutella (02/08/25)	22,687	54,705	4.8	34	17	[1,66]	297	[0,0]	No
P2P networks	Gnutella (02/08/24)	26,518	65,369	4.9	34	17	[1,355]	297	[0,44]	No
1 21 notworks	Gnutella (02/08/30)	36,682	88,328	4.8	35	18	[1,55]	314.5	[35,70]	No
	Gnutella (02/08/31)	62,586	147,892	4.7	37	19	[1,95]	351	[39,76]	No
 	Amazon (03/03/02)	262,111	899,792	6.9	42	21	[1,420]	451	[88,132]	No
Amazon	Amazon (2012)	334,863	925,872	5.5	43	22	[1,549]	472.5	[0,0]	No
Co-purchasing	Amazon (03/03/12)	400,727	2,349,869	11.7	43	22	[1,2747]	472.5	[52,285]	No
Networks	Amazon (03/06/01)	403,394	2,443,408	12.1	43	22	[1, 2752]	472.3	[52, 333]	No
INCLINUINS	Amazon (03/05/05)	410,236	2,443,408	11.9	43	22	[1,2760]	472.5	[50,333]	No
Road	Pennsylvania	1,088,092	1,541,898	2.8	43	24	[1,2760]	563.5	[0,0]	No
Networks	Texas	1,379,917	1,921,660	2.8	47	24	[1,12]	563.5	[0,0]	No
INCLWOIKS	California			2.8	49	25		612		No No
	Camornia	1,965,206	2,766,607	2.8	49	25	[1, 12]	012	[0, 0]	INO

and the resulting G becomes G^{W_i} .

Let G^{W_l} denote a watermarked graph for user l ($l \neq i$), built using a different seed Ω_l . Then with low probability, any subgraph of G^{W_l} or G is isomorphic to S^{W_i} .

PROOF. We first show that with low probability, any subgraph of G^{W_l} is isomorphic to S^{W_i} . Let $Y = \{y_1, y_2, ... y_k\}$ be a set of ordered nodes in G^{W_l} , where each y_i maps to a node x_i in X. We define an event \mathcal{E}_Y occurs if the subgraph $G^{W_l}[Y]$ is isomorphic to $G^{W_i}[X]$ or S^{W_i} . Then the event \mathcal{E} representing the fact that there exists at least one subgraph on G^{W_l} that is isomorphic to S^{W_i} is the union of events \mathcal{E}_Y on all possible Y, i.e. $\mathcal{E} = \bigcup_Y \mathcal{E}_Y$.

Next, we compute the probability of event \mathcal{E} by those of individual event \mathcal{E}_Y . Specifically, we first show that the probability of an edge exists between node x_i and x_j

 $(j \neq i+1)$ in $S^{W_i} = G^{W_i}[X]$ is $\frac{1}{2}$. This is because each edge in the random graph W_i is independently generated with probability $\frac{1}{2}$. After performing the XOR operation between W_i and S, the probability of an edge exists between x_i and x_j $(j \neq i+1)$ on S^{w_i} is $\frac{1}{2} \cdot p_{ij} + (1-p_{ij}) \cdot \frac{1}{2} = \frac{1}{2}$ where p_{ij} is the probability that an edge exists between x_i and x_j on S. Thus the result of XOR between W_i and S is also a random graph, and its edge generation is independent of that in G^{W_i} , $l \neq i$. Furthermore, it is easy to show that our design applies XOR operations on $\binom{k}{2} - (k-1)$ node pairs on the k nodes, and each node pair has an edge with a probability of $\frac{1}{2}$. Thus, the probability of a subgraph $G^{W_i}[Y]$ being isomorphic to S^{W_i} is $P(\mathcal{E}_Y) = \frac{1}{2}^{\binom{k}{2}-(k-1)} \cdot \beta$ where $\beta \leq 1$ is the probability that every (y_i, y_{i+1}) pair in $G^{W_i}[Y]$

is connected. Thus $P(\mathcal{E}_Y) \leq \frac{1}{2}^{\binom{k}{2}-(k-1)}$.

Since $\mathcal{E} = \bigcup_Y \mathcal{E}_Y$ and there are less than n^k possible sets of k ordered nodes in G^{W_l} , we use the Union Bound to compute the probability of event \mathcal{E} as follows:

$$P(\mathcal{E}) < n^k \cdot P(\mathcal{E}_Y) \le n^k \cdot \frac{1}{2} \frac{\binom{k}{2} - (k-1)}{2}$$

$$= 2^{\frac{k^2}{2+\delta}} \cdot \frac{1}{2}^{\frac{k^2 - 3k}{2} + 1} = \frac{1}{2}^{\frac{\delta k^2}{2(2+\delta)} - \frac{3k}{2} + 1}$$
(2)

The above equation shows that the probability $P(\mathcal{E})$ reduces exponentially to 0 as k increases.

Finally, we can apply the same method to show that with low probability, any subgraph of G is isomorphic to S^{W_i} . This is because the XOR operations between W_i and S produce a random graph that is independent of G. This concludes our proof. \square

5.2 Watermark Detectability

In addition to providing uniqueness, a practical watermark design should also offer low detectability, *i.e.* with low probability each watermark gets identified by external users/attackers. This means that without knowing the seed Ω_i associated with user i, the embedded watermark graph S^{W_i} should not be easily distinguishable from the rest of the graph G^{W_i} . Therefore, the detectability would depend heavily on the topology of the original graph G, *i.e.* a watermark graph can be well hidden inside a graph G^{W_i} if its structural property is not too different from that of G.

In the following, we examine the detectability of watermarks in terms of a graph's suitability for watermarking. This is because directly quantifying the detectability is not only highly computational expensive¹, but also lacks a proper metric. Instead, we cross-compare the key structural properties of S^{W_i} and G, and define G as being suitable for watermarking if its structure properties are similar to that of S^{W_i} , implying a low watermark detectability.

Suitability for Watermarking. To evaluate a graph's suitability for watermarks, we first study the key structure property of the embedded watermark graph S^{W_i} . To guarantee watermark uniqueness and minimize distortion, the watermark graph S^{W_i} needs to be a random graph with an edge probability of $\frac{1}{2}$ (except for the fixed edges between x_i, x_{i+1} node pairs), and include $k = (2+\delta)\log_2 n$ nodes. Thus its average node degree is at least (k+1)/2 and its average graph density is $\binom{k}{2} + k - 1/2$.

Table 2: Size and density of subgraph on nodes with degree > (k+1)/2 in each graph. Size is the number of subgraph nodes, and density is quantified as average edges each node having inside the subgraph.

ach node havin	Sub					
Graph	Node #	Avg. Deg.	k	ermark Graph Avg. Deg.	Suitability	
Russia	4.794	22.2	κ 39	20.0	Yes	
L.A.	196,174	49.2	45	23.0	Yes	
L.A. London	562,075	56.1	48	24.5	Yes	
			38			
Epinions (1)	7,083	68.7		19.5	Yes	
Slashdot (08/11/06)	9,908	53.4	38	19.5	Yes	
Twitter	34,014	60.5	38	19.5	Yes	
Slashdot (09/02/16)	10,065	53.0	38	19.5	Yes	
Slashdot (09/02/21)	10,105	53.2	38	19.5	Yes	
Slashdot (09/02/22)	10,605	53.4	38	19.5	Yes	
GPlus	68,828	347.1	39	20.0	Yes	
Epinions (2)	10,363	83.5	40	20.5	Yes	
Youtube	31,720	45.1	47	24.0	Yes	
Pokec	564,001	53.0	48	24.5	Yes	
Flickr	136,202	174.5	48	24.5	Yes	
Livejournal	945,567	57.5	52	26.5	Yes	
Patents	2,370	15.6	34	17.5	Yes	
ArXiv (Theo. Cit.)	12,054	43.4	34	17.5	Yes	
ArXiv (Phy. Cit.)	14,785	37.9	35	18.0	Yes	
ArXiv (Phy.)	2,860	62.5	32	16.5	Yes	
ArXiv (Astro)	6,536	42.9	33	17.0	Yes	
DBLP	15,004	17.3	43	22.0	Yes	
ArXiv (Condense)	178,455	16.0	51	26.0	Yes	
Email (Enron)	3,481	48.2	35	18.0	Yes	
Email (Europe)	1,779	44.0	42	21.5	Yes	
Wiki Talk	21,253	83.1	49	25.0	Yes	
Stanford	35,600	42.1	42	21.5	Yes	
NotreDame	16,831	38.7	43	22.0	Yes	
BerkStan	110,202	57.0	45	23.0	Yes	
Google	55,431	14.8	46	23.5	Yes	
Brightkite	4,586	30.8	37	19.0	Yes	
Gowalla	17,946	39.3	41	21.0	Yes	
Oregon (1)	264	17.1	31	16.0	Yes	
Oregon(2)	579	31.0	32	16.5	Yes	
CAIDA	575	16.0	34	17.5	Yes	
Skitter	146,601	50.0	48	24.5	Yes	
Gnutella (02/08/04)	796	5.2	31	16.0	No	
Gnutella (02/08/25)	499	2.0	34	17.5	No	
Gnutella (02/08/24)	709	2.7	34	17.5	No	
Gnutella (02/08/30)	1,001	3.8	35	18.0	No	
Gnutella (02/08/31)	1,276	3.6	37	19.0	No	
Amazon (03/03/02)	3,727	2.8	42	21.5	No	
Amazon (2012)	5,318	2.5	43	22.0	No	
Amazon (03/03/12)	25,717	6.7	43	22.0	No	
Amazon (03/06/01)	28,081	7.3	43	22.0	No	
Amazon (03/05/05)	28,044	7.5	43	22.0	No	
Pennsylvania	0	0	47	24.0	No	
Texas	0	0	47	24.0	No	
California	0	0	49	25.0	No	

Given these properties of the embedded watermark, we note that watermark node degree and density can be higher than those of many real-world graphs, such as those listed in Table 1. Intuitively, to ensure low detectability of such a watermark graph, suitable graphs should include a set of nodes (D) which are difficult to distinguish from the watermark nodes in term of node degree and subgraph density. Specifically, a suitable graph dataset needs to contain a set of nodes D with degree comparable or higher than the watermark graph node degree; and the density of the subgraph on D is at least comparable to the watermark graph density. If these two properties hold, the embedded watermark

¹Each embedded watermark graph is similar to a random graph with $\frac{1}{2}$ edge probability. Thus the detectability is low if certain subgraphs of G are also random graphs with similar edge probabilities. Yet identifying these subgraphs (and the embedded watermark graph) on a large graph incurs significant computation overhead.

graph cannot be easily distinguished from D in the graph, and therefore cannot be detected by attackers.

To capture the above intuition, we define that a graph G is suitable for watermarking if its node degree and graph density satisfy the following two criteria. First, the minimum and maximum node degree of G, denoted as $N_{min}(G)$ and $N_{max}(G)$ respectively, need to satisfy $N_{min}(G) \leq (k+1)/2 \leq N_{max}(G)$. Second, across all k-node subgraphs of G whose node degree expectation is greater than (k+1)/2, the minimum and maximum graph density need to satisfy $D_{min}(k) \leq {k \choose 2} + k - 1/2 \leq D_{max}(k)$. Together, these two criteria ensure that the embedded watermark graph can be "well hidden" inside G^{W_i} .

To compute $D_{min}(k)$ and $D_{max}(k)$, we need to enumerate all possible subgraphs of G, which is computationally prohibitive for large graphs. Thus we apply a sampling method to estimate them. To estimate $D_{max}(k)$, we identify the subgraph with the highest density using a greedy search: starting from a randomly chosen node v_1 with degree > (k+1)/2, pick the 2nd node v_2 with degree > (k+1)/2that is connected to v_1 , then the 3rd node v_3 with degree > (k+1)/2 who has the most number of edges to v_1 and v_2 . This greedy search stops until we find k nodes. We repeat the same process for all the nodes with degree > (k+1)/2, creating multiple subgraphs from which we calculate the density and pick the highest one. To estimate $D_{min}(k)$, we apply a similar process to locate multiple subgraphs except that for each subgraph we locate the next node v_{i+1} randomly as long as its node degree > (k+1)/2 and it connects to at least one of the existing nodes $\{v_1, ... v_i\}$.

Suitability of Real Graph Datasets. We wanted to understand how restrictive our suitability constraints were in the context of real graph datasets available today. We consider 48 real network graphs ranging from 10K nodes, 39Kedges to 5M nodes and 48M edges. These graphs represent vastly different types of networks and a wide range of structural topologies. They include 3 social graphs generated from Facebook regional networks matching Russia, L.A., and London [41]. They include 12 other graphs from online social networks, including Twitter [21], Youtube [44], Google+ [21], Slovakia Pokec [39], Flickr [26], Livejournal [26], 2 snapshots from Epinions [33], and 4 snapshots from Slashdot [19]. We also add 3 citation graphs from arXiv and U.S. Patents [15], 4 graphs capturing collaborations in arXiv [15] and DBLP [44], 3 communication graphs generated from 2 Email networks [17, 19] and Wiki Talk [20], 4 web graphs [18, 2], 2 location-based online social graphs from Brightkite and Gowalla [6], 5 snapshots of P2P file sharing graph from Gnutella [17], 4 Internet Autonomous System (AS) maps [15], 5 snapshots of Amazon co-purchasing networks [16, 44], and 3 U.S. road graphs [18]. The statistics of all graphs are listed in Table 1.

For all graphs, we use $\delta=0.3$ to ensure a 99.999% watermark uniqueness, and compute and list the corresponding value of k (from Equation 2) in Table 1. Next we list the

two criteria in terms of (k+1)/2 vs. $[N_{min}(G), N_{max}(G)]$, and $\binom{k}{2} + k - 1/2$ vs. $[D_{min}(k), D_{max}(k)]$. If a graph satisfies both criteria, our analytical results will hold for any watermarks embedded on it.

We can make two observations based on results from Table 1. *First*, 35 out of our 48 total graphs are suitable for watermarking. Also note that graphs describing similar networks are consistent in their suitability. For example, all 15 graphs from various online social networks are suitable for watermarks. *Second*, all 13 graphs unsuitable for watermarks come from only 3 kinds of networks, *i.e.* Amazon copurchasing networks, P2P networks, and Road networks. These results in each group are self consistent. These results support our assertion that our proposed watermarking mechanism is applicable to most of today's network graphs with low detection risk. In practice, the owner of a graph can apply the same mechanism to determine if her graph is suitable for our watermark scheme.

To understand key properties determining whether a graph is suitable for watermarking, we measure various graph structrual properties, including average node degree, node degree distribution, clustering coefficient, average path length, and assortativity. We also consider the size and density of subgraphs on nodes with degree more than watermark minimum average degree (k+1)/2. Our measurement results show that the size and density of subgraphs on nodes with degree > (k+1)/2 are the most important properties to determine suitability. Here, the size of these subgraphs is the number of nodes in the subgraph, and the density of the subgraph is measured as the average edges each node has inside the subgraph, i.e. average degree inside the subgraph. As shown in Table 2, unsuitable graphs do not have subgraphs with density to comparable to watermarks, while subgraphs with the desired density can be found in graphs deemed suitable. These results are consistent with our intuition on quantifying suitability of watermarks.

Summary. Since the average watermark subgraph has high node degree and density, a graph suitable for watermarking must include a set of nodes, whose degree and subgraph density are comparable or even higher than watermark subgraphs. We propose two criteria targeting at node degree and subgraph density respectively to quantify whether a graph is suitable for watermarking. We collect a large set of available graph datasets today, and find that 35 out of 48 real graphs are suitable for watermarking. This promising result indicates that watermark technique can be applied on most of real networks with low probability to be identified.

6. MORE ROBUST WATERMARKS

Our basic design provides the fundamental building blocks of graph watermarking with little consideration of external attacks. In practice, however, malicious users can seek to detect or destroy watermarked graphs. Here, we first describe external attacks on watermarks, and then present advanced features that defend against the attacks. Note that these improvement techniques aim to increase the cost of attacks rather than disabling them completely. Finally, we reevaluate the watermark uniqueness of the advanced design.

6.1 Attacks on Watermarks

As discussed earlier, our attack model includes attacks trying to destroy watermarks while preserving the topology of the original graph. Based on the number of attackers, attacks on watermarks fall under our two attack models: single attacker and multiple colluding attackers. With access to only one watermarked graph, a single attacker can modify nodes and/or edges in the graph to destroy watermarks. With multiple watermarked graphs, colluding attackers can perform more sophisticated attacks by cross-comparing these graphs to detect or remove watermarks.

Single Attacker Model. The naive edge attack is easiest to launch, and tries to disrupt the watermark by randomly adding or removing edges on the watermarked graph. For the attacker, there is a clear tradeoff between the severity of the attack (number of edges or nodes modified), and the structural change or distortion applied to the graph structure.

At first glance, this attack seems weak and unlikely to be a real threat. The probability of the attacker modifying one edge or node in the embedded watermark graph W_i is extremely low, given the relatively small size of W_i compared to the graph. As shown later, however, this attack can be quite disruptive in practice. By modifying a node n_i or an edge connected to n_i , the attack impacts all of n_i 's neighboring nodes, since their NSD labels will be modified. These NSD label changes, while small, are enough to make locating nodes in the watermark graph very difficult. This effect is exacerbated in social graphs that exhibit a small world structure, since any change to a supernode's degree will impact a disproportionately large portion of nodes in the graph.

One extreme of this attack is to leak patial watermarked graphs or merge several graphs together. With high probability, it can destroy the embedded watermarks, but will significantly distort the graph topologies to reduce their usability. Thus, we do not consider such scenarios in our study.

Collusion Attacks. By obtaining multiple watermarked graphs, an attacker can compare these graphs to eliminate watermarks. Since we anonymize each watermarked graph by randomly reassigning node IDs (see Section 4.1), attackers cannot directly match individual nodes across graphs. To compare multiple graphs, we apply the deanonymization methods proposed in [27, 28]. Specifically, we first match 1000 highest degree nodes between two graphs based on their degree and neighborhood connectivities [28], and then start from these nodes to find new mappings with the network structure and the previously mapped nodes [27].

Using the deanonymization method, attackers can then build a "cleaned" graph, where an edge exists if it exists in the majority of the watermarked graphs. Since embedded watermark graphs are likely embedded at different locations on each graph, a majority vote approach effectively removes the contributions from watermark subgraphs, leading to a graph that closely approximates the original G.

6.2 Improving Robustness against Attacks

The attacks discussed above can disrupt the watermark extraction process in two ways. First, adding or deleting nodes/edges in G' changes node degrees, and therefore nodes' NSD labels, thereby disrupting the identification of candidate nodes during the second step of the extraction process; second, adding or deleting nodes/edges inside the embedded watermark graph S^{Wi} can change the structure of the watermark graph, making it difficult to identify during the third step of the extraction process. To defend against these attacks, we must make the watermark extraction process more robust against attack-induced artifacts on both node and graph structure. To do so, we propose four improvements over the basic extraction design in Section 4.2.

Improvements #1, #2: Addressing changes to node neigh-Extracting a watermark involves searching borhoods. through nodes in G' by their NSD labels. By adding or deleting nodes/edges, attackers can effectively change NSD labels across the graph. To address this, we propose two changes to the basic extraction design. First, we bucketize node degrees (with bucket size B) to reduce the sensitivity of a node's NSD label to its neighbors' node degrees. For example, with B = 5, a node with degree 9 will stay in the same bucket even if one of its edges has been removed (reducing its node degree to 8). Second, when selecting a watermark node's candidate node list, we replace the exact NSD label matching with the approximate NSD label matching. That is, a match is found if the overlap between two bucketized NSD labels exceeds a threshold θ . For example, with $\theta = 50\%$, a node with bucketized NSD label "1-2-3-4" would match a node with label "1-2-3" since the overlap is 75% > θ .

These changes clearly allow us to identify more candidate nodes for each watermark node, thus improving robustness against small local modifications. On the other hand, more candidate nodes lead to more computation during the subgraph matching step, *i.e.* step 3 in the extraction process. Such expansion, however, does not affect watermark uniqueness and detectability, since they are unrelated to the size of candidate pools.

Improvement #3, #4: Addressing changes to subgraph structure. Random changes made to G' by an attacker has some chance of directly impacting a node or edge in the embedded watermark. To address this, we propose two techniques. First, we add redundancy to watermarks by embedding the same watermark graph W_i into m disjoint subgraphs $S_1, S_2, ...S_m$ from the original graph G. This greatly increases the probability of the owner locating at least one unmodified copy of W_i during extraction, even in the presence of attacks that make significant changes to nodes and edges in G'. Note that since we embed watermarks on disjoint subgraphs, this does not affect watermark uniqueness

 $1-P(\mathcal{E})$. While embedding m watermarks will impact false positive, which is $1-(1-P(\mathcal{E}))^m$.

Second, it is still possible that all the watermark graphs are "destroyed" by the attacker and there are no matches in the extraction process. If this happens, we replace the exact subgraph matching in the step 3 of the extraction process with the approximate subgraph matching. That is, a subgraph matches the watermark graph if the amount of edge difference between the two is less than a threshold L. By relaxing the search criteria used in step 3 of the extraction process, this technique allows us to identify "partially" damaged watermarks, thus again improving robustness against attacks. However, it can also increase false positives in watermark extraction, reducing watermark uniqueness. We show later in this section that the impact on watermark uniqueness can be tightly bounded by controlling L.

Improvement #5: Addressing Collusion Attacks. Recall that for powerful attackers able to match graphs at an individual node level, they can leverage majority votes across multiple watermarked graphs to remove watermarks. To defend against this, our insight is to embed watermarks that have some portion of spatial overlap in the graph, such that those components will survive majority votes over graphs.

We propose a hierarchical watermark embedding process to protect watermark(s) against collusion attacks. To build watermarked graphs for M users, we uniform-randomly divide these M users into 2 groups $(a_1 \text{ and } a_2)$ and associate each group with a public-private key pair $< K_{pub}^{a_1}, K_{priv}^{a_1} >$ or $< K_{pub}^{a_2}, K_{priv}^{a_2} >$, which is generated and held by the data owner. We repeat this to produce another group partition and randomly divide M users into 2 groups $(b_1 \text{ and } b_2)$ associated with group key pairs $< K_{pub}^{b_1}, K_{priv}^{b_1} >$ and $< K_{pub}^{b_2}, K_{priv}^{b_2} >$ separately. After this step, each user is assigned to two groups. For example, a user i is assigned to groups a_1 and b_2 .

To prevent the data owner or users from forging group assignments, we modify step 1 in Section 4.1 to achieve an agreement on group assignments between the data owner and each user. More specifically, at time T when the data owner tends to share its graph with a user i assigned to two groups, e.g. groups a_1 and b_2 , the data owner first send user i three items: current timestamp T and two group signatures $K^{a_1}_{priv}(T)$ and $K^{b_2}_{priv}(T)$. User i then validates the two group signatures using the two group public keys $K^{a_1}_{pub}$ and $K^{b_2}_{pub}$. If the timestamps encrypted using group private keys are T, user i agrees the group assignment, saves the three items, and sends back its personal signature, i.e. $K_{vriv}^{i}(T)$; otherwise, user i rejects the group assignment. Once the data owner receives user i's signature $K_{priv}^i(T)$, it validates this timestamp with user i's public key. If it is valid, the data owner generates three seeds for user i: Ω_i by combining $K^i_{priv}(T)$ and K^G , Ω_{a_1} by combining $K^{a_1}_{priv}$ and K^G , and Ω_{b_2} by combining $K_{priv}^{b_2}$ and K^G , where K^G is graph key for graph G. Through this agreement scheme, either the data owner or users can not forge their group assignments. Moreover, since the generated seed for each group is unique, we can make sure that only one unique watermark corresponds to each group.

To embed the watermarks for user i, we first follows step 2-4 in Section 4.1 to embed two group watermarks using its two group seeds generated through the above method, i.e. Ω_{a_1} and Ω_{b_2} in the example. We then use user i's individual seed, i.e. Ω_i , to embed an individual watermark. When generating the group watermarks, we make sure that 1) the group watermark remains the same for users in the same group; and 2) watermarks corresponding to different groups do not overlap with each other, or with each user's individual watermark graph. Note that because the group and individual watermarks are generated with different seeds, this hierarchical embedding process does not affect watermark uniqueness.

Under this design, a collusion attack can successfully destroy all the watermarks (group or individual) only if the majority of the watermarked graphs come from different user groups. Otherwise, the majority vote on raw edges will preserve the "group watermark." We can compute the success rate of the attack by the following equation, which represents the probability that the majority of the graphs obtained by the attacker come from different user groups:

$$\lambda(M_a, J) = \left(1 - J \sum_{i = \lceil \frac{M_a + 1}{2} \rceil}^{M_a} {M_a \choose i} \cdot \left(\frac{1}{J}\right)^i \cdot \left(\frac{J - 1}{J}\right)^{M_a - i}\right)^2$$

where M_a is the number of watermarked graphs obtained by the attacker and J is the number of groups in each group partition. The above design chose J=2 because it minimizes $\lambda(M_a,J), \forall M_a$. Furthermore, when M_a is odd, $\lambda(M_a,2)=0$; and when M_a is even, $\lambda(M_a,2)$ is at most 0.25 when $M_a=2$. Note that in equation (3) the operation $(.)^2$ is due to the fact that we group the users twice into two different group classes: a_1,a_2 and b_1,b_2 . If we only perform the group partition once (e.g. dividing the users into a_1,a_2), then $\lambda(2,2)=0.5$. This means that in practice we can further reduce λ by performing multiple rounds of group division (2 in the above design) and adding more group watermarks.

Note that group watermarks contain much less information than single user watermarks. In fact, the more robust a group watermark, the larger granularity (and less precision) it will provide. Our proposed solution is to extend the system by using additional "dimensions," e.g. go beyond the two dimensions of a and b mentioned above. Combining results from multiple dimensions will quickly narrow down the set of potential users responsible for the leak. However, since a colluding attack requires the involvement of multiple leakers, even identifying a single leaker is insufficient. Developing a scheme to reliably detect multiple (ideally all) colluding users is a topic for future work.

6.3 Impact on Watermark Uniqueness

To improve the robustness of our watermark system, we relax the subgraph matching criteria from exact matching to approximate matching with at most L edge difference. Such relaxation does not affect watermark detectability because it does not change the embedding process. However, it may affect watermark uniqueness, which we will analyze next.

Consider two watermarked graphs G^{W_i} and G^{W_j} that were independently generated for user i and j following the three steps defined in Theorem 1. Let S^{W_i} and S^{W_j} represent the embedded watermark graph in G^{W_i} and G^{W_j} , respectively. To examine the watermark uniqueness, we seek to compute the probability that a subgraph in G^{W_j} differs from S^{W_i} by at most L edges.

Our analysis follows a similar structure of Theorem 1's proof. Let \mathcal{E}_Y denote the event where a subgraph of G^{W_j} built on k nodes $Y = \{y_1, y_2, ..., y_k\}$ only differs from S^{W_i} by at most L edges. Our goal is to calculate the probability of the event $\mathcal{E} = \bigcup_Y \mathcal{E}_Y$, which is the union on all combinations of k nodes. To do so, we first compute the probability of individual \mathcal{E}_Y .

As shown in Theorem 1, the edges between $\binom{k}{2}-(k-1)$ node pairs in S^{W_i} are generated randomly with probability $\frac{1}{2}$ and are independent of G^{W_j} , while the rest k-1 edges ($< x_l, x_{l+1} >, l=1...k-1$) are fixed. Thus we can show that the probability that a subgraph $G^{W_j}[Y]$ differs from S^{W_i} by h edges is upper bounded by $\frac{1}{2}^{e-k+1} \cdot \binom{e}{h}$ where $e=\binom{k}{2}$. Therefore, we can derive the probability of \mathcal{E}_Y as $P(\mathcal{E}_Y) \leq \frac{1}{2}^{e-k+1} \cdot \sum_{h=0}^{L} \binom{e}{h}$. And consequently, we have

$$P(\mathcal{E}) \le n^k \cdot \frac{1}{2}^{e-k+1} \cdot \sum_{h=0}^{L} \binom{e}{h} \tag{4}$$

where $e = {k \choose 2}$, $k = (2 + \delta)log_2 n$, and n is the node count of G^{W_j} .

Next, given the probability of uniqueness $1-P(\mathcal{E})$, we compute the upper bound on L to ensure $1-P(\mathcal{E}) \geq 0.99999$ for all the graphs in Table 1 except Road graphs, Amazon graphs and P2P network graphs. Again we set $\delta=0.3$. The result is listed in Table 3, where the maximum limit of L varies between 0 and 12. In general, the larger the graph, the higher the upper bound on L.

7. EXPERIMENTAL EVALUATION

We evaluate the proposed graph watermarking system using real network graphs. We consider three key performance metrics, *false positive*, *graph distortion* and *watermark robustness*. Having analytically quantified the watermark uniqueness in Section 5 and 6, we focus on examining graph distortion and watermark robustness while ensuring false positive less than 0.001%. We also study the computational efficiency of the proposed watermark embedding and extraction schemes.

Experiment Setup. Given the large number of graph

Table 3: Upper bound of L for the 35 network graphs.

Graph	Oregon (1)	Oregon (2)	CAIDA	Email	arXiv	
Grapii	Olegon (1)		CAIDA	(Enron)	(Theo. Cit.)	
L Bound	0	1	1	1	1	
Graph	arXiv	arXiv	arXiv	Patent	Slashdot	
Grapii	(Phy. Cit.)	(Phy.)	(Astro)	1 atent	(08/11/06)	
L Bound	1	1	1	2	3	
Graph	Twitter	Slashdot	Slashdot	Slashdot	Brightkite	
Grapii	TWITTE	(09/02/16)	(09/02/21)	(09/02/22)	Diigitkite	
L Bound	3	3	3	3	3	
Graph	Russia	Epinions (1)	Google+	Epinions (2)	Standford	
L Bound	4	4	4	5	5	
Graph	Email	Gowalla BerkStand		DBLP	NorteDame	
Grapii	(Europe)	Gowana	Derkständ	DBLI	NoneDame	
L Bound	5	5	6	7	7	
Graph	L.A.	London	Flickr	Wiki	Google	
L Bound	8	8	8	8	8	
Graph	Skitter Youtub		e Pokec	arXiv	Livejournal	
Graph	Skiller	Toutube	FOREC	(Condense)	Livejournai	
L Bound	8	9	9	11	12	
	•	•	•	•	•	

Table 4: Percentage of modified nodes and edges after embedding 5 watermarks into a graph and impact on graph structure (dK-2 Deviation).

Graph	Nodes (%)	Edges (%)	dK-2 Deviation
Watermarked LA	0.037%	0.033%	0.0008
Watermarked Flickr	0.014%	0.019%	0.0001

computations per data point, we focus our experiments on two of the larger network graphs listed in Table 1, the LA regional Facebook graph and the Flickr network graph. The two graphs have very different sizes and graph structures. To guarantee less than 0.001% false positive, we select $\delta=0.3$, and the k values for the LA and Flickr graphs are 45 and 48, respectively. For our basic design, we generate 1 watermark per graph. For our advanced design, we set L to 8, the degree bucket size to 10, and the NSD similarity threshold to $\theta=0.75$. For each user, we embed 5 watermarks in its graph, 3 as individual watermarks and 2 as group watermarks. We chose these settings because they are intuitive and work well in practice. We leave the optimization of these parameters to future work.

In the following, we present our experiment results in terms of 1) amount of distortion introduced to the original graph due to watermarking, 2) robustness of the watermark against attacks, and 3) computational efficiency of our watermarking design.

7.1 Graph Distortion from Watermarks

We consider three types of metrics for measuring the graph distortion from watermarks.

- Modifications to the raw graph We count the number of nodes and edges modified by embedding watermarks. Intuitively, more modifications to the graph introduce higher distortion.
- Deviation in the dK-2 distribution We also measure the Euclidean distance between the dK-2 series of the original

Table 5: Graph metrics are consistent w/ and w/o water-marks.

Graph	AS	Avg. CC	Avg. Deg	Avg. Path	Dia.
LA Original	0.21	0.19	25.4	4.6	14
Watermarked	0.21	0.19	25.4	4.6	14
Flickr Original	-0.02	0.18	18.1	5.3	21
Watermarked	-0.02	0.18	18.1	5.3	21

graph and that of the watermarked graphs². Larger deviation in dK-2 series implies higher distortion to the graph structure.

Graph metrics w/ and w/o watermarks – Finally we measure the commonly used graph metrics before and after the watermarking, including node degree distribution, assortativity (AS) [36], clustering coefficient (CC) [36], average path length and diameter. Any large deviation in any of these metrics indicates that the watermarked graph experienced large distortion.

We have examined the distortion introduced by both the basic and advanced designs. We only show the results of the advanced design because it adds more watermarks and thus leads to higher distortion. For both LA and Flickr graphs, we generate 10 different watermarked graphs (using 10 different random generator seeds) and present the average result across these graphs. Because computing average path length and diameter on these two large graphs is highly computational intensive, we randomly sample 1000 nodes and compute the average path length and diameter among them (following the same approach taken by prior works on social graph analysis [41]).

Table 4 shows the percentage of modified nodes and edges by watermarking. Even after embedding 5 watermarks, the modification is less than 0.04% for LA and 0.02% for Flickr. These small changes imply little distortion on the watermarked graphs. This is further confirmed by the average dK-2 distances for both graphs, 0.0008 for LA and 0.0001 for Flickr, indicating that the watermarked graphs are highly similar to the original graph.

Table 5 compares the original and watermarked graphs in terms of five representative graph metrics. Similarly, for both LA and Flickr, the graph metrics remain the same before and after watermarking. We also examined the statistical distribution of each metric and found no visible difference between the graphs.

Together, these results indicate that our proposed watermarking system successfully embeds watermarks into graphs with negligible impact on graph structure. This is unsurprising, given the extremely small size of watermarks relative to the original graphs. Thus we believe watermarked graphs can replace the originals in graph applications and produce (near-)identical results.

7.2 Robustness against Attacks

Next, we investigate how the proposed watermarking system performs in the presence of attacks. For each of the two attack implementations discussed in Section 6.1, we vary the attack strength and examine the robustness against the attack as well as the cost of the attack. Specifically, we repeat each experiment for 10 times, and examine two metrics:

- Robustness in the single attacker model, the robustness is quantified as the ratio of graphs from which we can successfully extract at least one of the 3 individual watermarks. In the collusion attack, in addition to this ratio, we also measure the ratio of graphs where we can extract at least one of the 5 watermarks (3 individual + 2 group watermarks).
- Cost of the attack the normalized distortion produced on the attacked graph. It represents the Euclidean distance between the dK-2 series of the attacked graphs and that of the original graph, normalized by the Euclidean distance between the dK-2 series of the clean watermarked graphs and that of the original graph. If the normalized distortion is larger than 1, the attack introduces more distortion than embedding the watermarks.

Results on the Single Attacker Model. For the single attacker model, we quantify the attack strength by the number of modified edges. The robustness and the cost of the attack are measured as a function of the number of modified edges.

To show how robustness is improved using the improvement mechanisms, we first evaluate the robustness results in the basic watermark scheme. We run the single attacker model on the watermarked graphs by varying the number of modified edge number, and repeat the experiment 10 times. The robustness here is quantified as the ratio of graphs from which we can successfully extract the watermark.

Figure 2 shows the robustness of the basic watermark scheme against the single attacker model. It shows that randomly modifying a small number of edges disrupted the watermark subgraph extraction process. In LA, our basic design cannot recover the watermark with 100% probability even when we modify 20 edges. In Flickr, a large graph, the robustness of the basic scheme reduces to less than 40%when only 500 edges are modified. In each case, at least one of the nodes in the watermark subgraph had a modified NSD label (one of its neighbors' node degree changed), and it could not be located in the extraction process. We also look at the distortion caused by the attack shown in Figure 3. As expected, the small number of modified edges causes small distortions in graph structures. For example, in LA, when the robustness is 0, the distortion is around 3x more than that caused by embedding the watermark. Both results show that watermarked graphs generated by the basic scheme are easily disrupted by even small, single user attacks.

Figure 4(a)-(b) plot the robustness of watermarked LA and Flickr graphs generated by the scheme with the improve-

The Euclidean distance between two dK-2 series G1 and G2 is defined by $\frac{1}{D}\sqrt{\sum_{< d_1,d_2>}(e^{G1}_{< d1,d2>}-e^{G2}_{< d1,d2>})^2}$ where D is the number of $< d_1,d_2>$ combinations or entries in the dK-2 series.

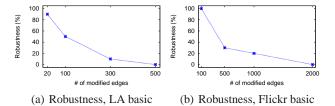


Figure 2: The robustness of the basic design against the single attacker model.

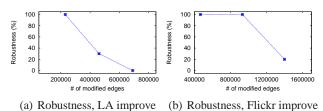


Figure 4: The robustness in the improved design against the single attacker model.

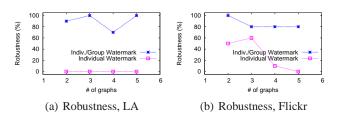


Figure 6: The robustness against the collusion attack.

ment mechanisms against the single attacker model. As expected, the robustness decreases with the attack strength since more edges are modified to "destroy" watermarks. For LA, our system maintains 100% robustness up to 230K modified edges, which is around 400x stronger than the maximum attack strength handled in the graph generated by the basic design. For Flickr, the system can handle attack strength up to 933K modified edges, which is > 400x stronger than the maximum attack strength in the basic design. This is because Flickr is larger in size while having a similar watermark graph size k, so the attacker must modify more edges to destroy watermarks. On the other hand, results in Figure 5 show that the cost of these attacks is large. For Flickr, with more than 1.4M modified edges, an attack leads to 800x more distortion over that caused by embedding the watermarks. Together, these results show that our watermark system with the improvement mechanisms is highly robust against single user attacks.

Results on Collusion Attacks. To implement the collusion attack desbribed in Section 6.1, we first generate 10 watermarked graphs and randomly pick M_a graphs from them as the graphs acquired by the attacker. We vary the number of graphs obtained by the attacker M_a between 2 to 5. For

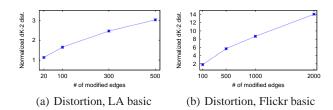


Figure 3: The distortion caused by the single attacker model in the basic design.

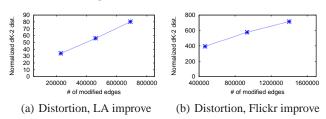


Figure 5: The distortion caused by the single attacker model in the improved design.

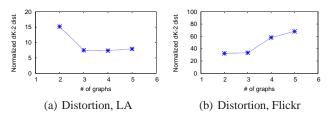


Figure 7: The distortion caused by the collusion attack.

each M_a value we repeat the experiments 10 times and report the average value. Since watermarks generated by the basic design can be easily detected and removed by the powerful collusion attack, here we focus on evaluating the robustness of the improvement mechanisms.

Figure 6(a)-(b) shows the robustness of the watermarked LA and Flickr graphs against the collusion attack. Figure 6(a) shows that in LA, by applying majority votes on raw edges, the collusion attack can effectively remove all 3 individual watermarks. However, the attack is ineffective in removing both group watermarks such that we can extract at least one group watermark in more than 60% of the attacked graphs. Here the robustness values, deviate slightly from that projected by Equation (3) because we limit the number of statistical sampling to 10 runs. Unlike LA, Figure 6(b) plots that the collusion attack cannot remove all the individual watermarks in Flickr when using 2 or 3 watermarked graphs. This is all because the deanonymization method causes a large portion of nodes mismatched in Flickr (30% nodes). Finally, Figure 7 shows that the collusion attack also introduce larger distortions in graph structure. This mainly comes from the mismatch of the deanonymization methods.

These results show that even a powerful collusion attack

is ineffective in removing all the embedded watermarks. Moreover, the potential inaccuracy of the deanonymization method causes the collusion attack even weaker in removing individual watermarks. Of course, the attackers will eventually succeed in disrupting watermarks if they are willing to modify larger portions of the graph, thus sacrificing the utility of the graph. While our work provides a robust defense against attackers with relatively low level of tolerance for graph distortion, we hope follow-on work will develop more robust defenses against higher distortion attacks.

7.3 Computational Efficiency

Here, we measure the efficiency of the watermarking system. There are two components in the watermarking system, *i.e.* watermark embedding and watermark extraction. The time to extract a watermark is the time to run step 2 and 3 in watermark extraction, *i.e.* candidate selection and watermark identification.

To accelerate the extraction process, we parallelize the key steps across multiple servers. More specifically, in the candidate selection step, any available servers are assigned an unchecked watermark node to find its candidates. In step 3, each available server will be assigned to search one watermark from one candidate of watermark node x_1 . When a watermark is found or no more candidates are unchecked, the extraction process stops (for that user).

We perform measurements to quantify the actual impact of parallelizing extraction over a cluster. All system parameters are the same as previous tests, except that we embed 1 watermark into a graph. To extract watermarks, we compare the improved watermark extraction method to the basic extraction method, with bucket size 10 and NSD similarity of 0.75 in the improved extraction method. In addition to the two graphs, *i.e.* Flickr and LA, we also measure efficiency on the largest graph in our study (Livejournal, 5.2 million nodes, 49 million edges), shown in Table 1. We parallelize watermark extraction across 10 servers, each with 2.33GHz Xeon servers with 192GB RAM. All experiments repeat on 10 different watermarked graphs, and the time is the average of the 10 computation time.

First result in Table 6 is that watermarking system is efficient in embedding and extracting watermarks. On average, embedding one watermark into a graph is very fast. For example, average embedding time for the largest graph, Livejournal, is around 12 minutes and embedding times for Flickr and L.A. are less than 2 minutes. Even using one server to extract watermarks, the computation time is small. Like in Flickr, the extraction time is around 13 minutes using both the basic method and the improved method. From our observation, the time to identify the watermark graph on the candidate subgraphs is much less than the time required to find and filter candidates, which corresponds to around 99% of total computation time. Since finding candidates takes O(kn) computational complexity and $k = (2+\delta)\log_2 n$, the complexity to extract a watermark from a real-world graph

Table 6: The efficiency of the watermarking system, including watermark embedding time on one server, the extraction time on one server and the parallel extraction time across 10 servers using basic watermark extraction method and improved watermark extraction method.

	Graph	Embedding (s)	Basic E	Extraction	Improved Extraction				
١	Grapii	Ellibedding (s)	Single(s)	Parallel (s)	Single (s)	Parallel (s)			
	LA	40	270	39	310	42			
	Flickr	80	767	195	776	197			
	Livejournal	695	2568	310	2605	317			

is $O(n \log_2 n)$. Here k is the number of nodes in the watermark graph and n is the number of nodes in the total graph.

Second, we find that speedup from distributed extraction is quite good, with speedup of 8 over 10 servers for Livejournal and 7 for LA (for both extraction methods). The speedup for Flickr is only around 4 using both methods, because one of the watermarked graphs takes much longer time than others in finding candidates, ~ 10 minutes. This is almost 4 times longer than the average extraction time on the other graphs. Not counting this outlier, average parallel extraction time on Flickr is around 150 seconds for both methods, which is 5 times faster than using one server. This is because the core computation is finding candidates, and completion time can vary when computing the similarity of NSD between watermark nodes and graphs nodes, which depends on node degree. The higher the degree is, the longer it takes for the similarity computation. Since there are several Flickr watermark nodes of high degree, time to find candidates is relatively longer.

Finally, comparing the two extraction methods, there is no significant difference between their computation time. This is because the extraction time of both methods are dominated by the time to find and filter candidates, which is $O(n\log_2 n)$ for both methods.

Summary. We evaluate the efficiency of the graph watermark embedding and extraction algorithms on three realworld graphs with $600K \sim 5M$ nodes and $7M \sim 48M$ edges. The results show that the embedding process is fast even for large graphs, and only takes up to 12 minutes to embed a watermark into a graph with 5M nodes. In the extraction process, the time to identify watermark graphs on the set of pre-filtered candidate nodes is much less than the time to filter candidate nodes, whose complexity is $O(n\log_2 n)$. Our experimental results also show that on a single commodity server, the extraction time is at most 43 minutes in a 5M-node graph, and can be future reduced to less than 5 minutes by distributing the computation across multiple servers.

8. CONCLUSION

In this paper, we take a first step towards the design and implementation of a robust graph watermarking system. Graph watermarks have the potential to significantly impact the way graphs are shared and tracked. Our work identifies the critical requirements of such a system, and provides an initial design that targets the critical properties of uniqueness, robustness to attacks, and minimal distortion to the graph structure. We also identify key attacks against graph watermarks, and evaluate them against an improved design with additional features for improved robustness under attack.

Our evaluation shows that our initial watermarking system modifies very few nodes and edges in a graph, *i.e.* less than 0.04% nodes and edges in a graph with 603K nodes and 7.6M edges. Results also demonstrate extremely low distortion, *i.e.* the watermarked graphs are highly consistent with the original graph in all graph metrics we considered. Empirical tests on several real, large graphs show that our robustness features dramatically improved our resilience against both single and multi-user collusion attacks. Finally, we show that the embedding process and the extraction process are efficient, and the extraction process is easily parallelized over a computing cluster.

While our proposed scheme achieves many of our initial goals, there is significant room for improvement and ongoing work. One focus is developing stronger redundancy schemes to protect against attackers with a greater tolerance for graph distortion, *i.e.* willing to make a greater number of node/edge changes. Another is to develop alternate schemes that can recover more information about multiple attackers in the colluding attack model.

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